88107202

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
HIGHER LEVEL
PAPER 2

Friday 5 November 2010 (morning)

2 hours

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer ***all*** *the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 6]*

Triangle ABC has $\text{AB} = 5\text{ cm}$, $\text{BC} = 6\text{ cm}$ and area 10 cm^2.

(a) Find $\sin\hat{B}$. *[2 marks]*

(b) **Hence**, find the two possible values of AC, giving your answers correct to two decimal places. *[4 marks]*

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2. *[Maximum mark: 4]*

The company *Fresh Water* produces one-litre bottles of mineral water. The company wants to determine the amount of magnesium, in milligrams, in these bottles.

A random sample of ten bottles is analysed and the results are as follows:

$$6.7,\ 7.2,\ 6.7,\ 6.8,\ 6.9,\ 7.0,\ 6.8,\ 6.6,\ 7.1,\ 7.3\ .$$

Find unbiased estimates of the mean and variance of the amount of magnesium in the one-litre bottles.

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3. *[Maximum mark: 5]*

The weight loss, in kilograms, of people using the slimming regime *SLIM3M* for a period of three months is modelled by a random variable X. Experimental data showed that 67 % of the individuals using *SLIM3M* lost up to five kilograms and 12.4 % lost at least seven kilograms. Assuming that X follows a normal distribution, find the expected weight loss of a person who follows the *SLIM3M* regime for three months.

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4. *[Maximum mark: 7]*

Find the equation of the normal to the curve $x^3y^3 - xy = 0$ at the point $(1, 1)$.

5. *[Maximum mark: 5]*

Solve the equations

$$\ln \frac{x}{y} = 1$$
$$\ln x^3 + \ln y^2 = 5.$$

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6. *[Maximum mark: 7]*

Consider the polynomial $p(x) = x^4 + ax^3 + bx^2 + cx + d$, where $a, b, c, d \in \mathbb{R}$.

Given that $1 + \mathrm{i}$ and $1 - 2\mathrm{i}$ are zeros of $p(x)$, find the values of a, b, c and d.

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7. *[Maximum mark: 6]*

The random variable X follows a Poisson distribution with mean m and satisfies

$$P(X=1)+P(X=3)=P(X=0)+P(X=2).$$

(a) Find the value of m correct to four decimal places. *[4 marks]*

(b) For this value of m, calculate $P(1 \le X \le 2)$. *[2 marks]*

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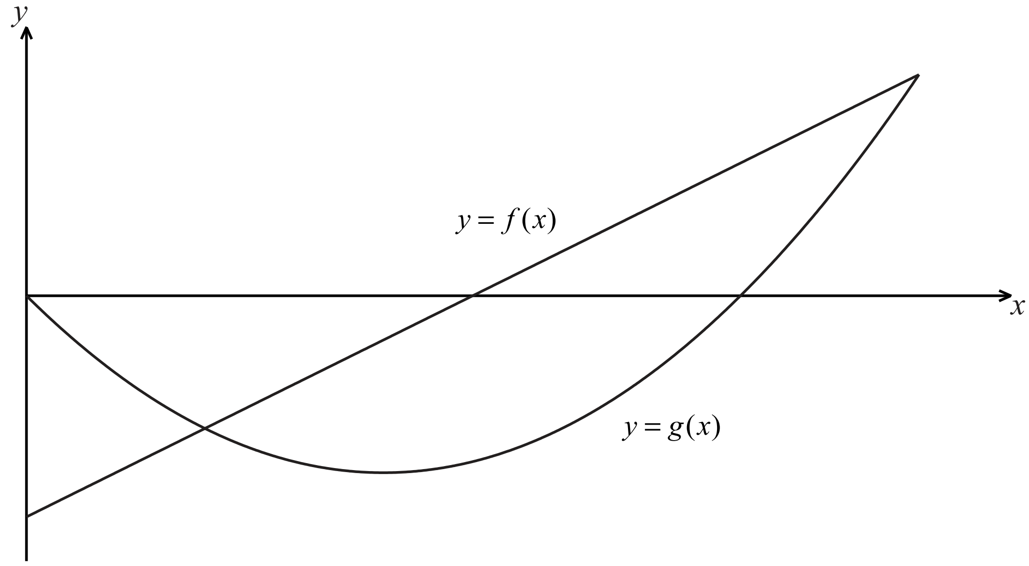
8. *[Maximum mark: 5]*

The diagram shows the graphs of a linear function f and a quadratic function g.



On the same axes sketch the graph of $\frac{f}{g}$. Indicate clearly where the x-intercept and the asymptotes occur.

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9. *[Maximum mark: 8]*

Consider the vectors $\boldsymbol{a} = \sin(2\alpha)\,\boldsymbol{i} - \cos(2\alpha)\,\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = \cos\alpha\,\boldsymbol{i} - \sin\alpha\,\boldsymbol{j} - \boldsymbol{k}$, where $0 < \alpha < 2\pi$.

Let θ be the angle between the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

(a) Express $\cos\theta$ in terms of α. *[2 marks]*

(b) Find the acute angle α for which the two vectors are perpendicular. *[2 marks]*

(c) For $\alpha = \frac{7\pi}{6}$, determine the vector product of $\boldsymbol{a}$ and $\boldsymbol{b}$ and comment on the geometrical significance of this result. *[4 marks]*

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10. *[Maximum mark: 7]*

The line $y = m(x - m)$ is a tangent to the curve $(1 - x)y = 1$.

Determine m and the coordinates of the point where the tangent meets the curve.

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*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

11. *[Maximum mark: 21]*

Tim throws two identical fair dice simultaneously. Each die has six faces: two faces numbered 1, two faces numbered 2 and two faces numbered 3. His score is the sum of the two numbers shown on the dice.

(a) (i) Calculate the probability that Tim obtains a score of 6.

(ii) Calculate the probability that Tim obtains a score of at least 3. *[3 marks]*

Tim plays a game with his friend Bill, who also has two dice numbered in the same way. Bill's score is the sum of the two numbers shown on his dice.

(b) (i) Calculate the probability that Tim and Bill **both** obtain a score of 6.

(ii) Calculate the probability that Tim and Bill obtain the same score. *[4 marks]*

(c) Let X denote the largest number shown on the four dice.

(i) Show that $\mathrm{P}(X \le 2) = \frac{16}{81}$.

(ii) Copy and complete the following probability distribution table.

x	1	2	3
$\mathrm{P}(X = x)$	$\frac{1}{81}$		

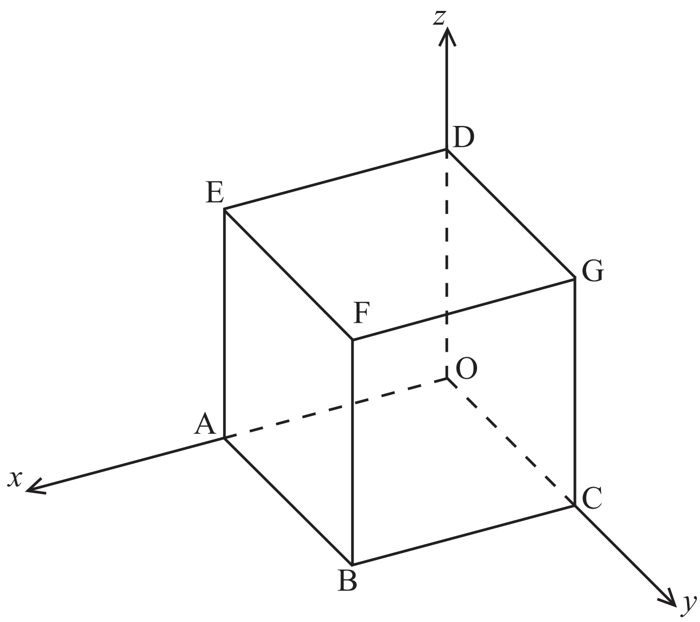
(iii) Calculate $\mathrm{E}(X)$ and $\mathrm{E}(X^2)$ and hence find $\mathrm{Var}(X)$. *[10 marks]*

(d) Given that $X = 3$, find the probability that the sum of the numbers shown on the four dice is 8. *[4 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

12. *[Maximum mark: 20]*

The diagram shows a cube OABCDEFG.



Let O be the origin, (OA) the x-axis, (OC) the y-axis and (OD) the z-axis.
Let M, N and P be the midpoints of [FG], [DG] and [CG], respectively.
The coordinates of F are $(2, 2, 2)$.

(a) Find the position vectors $\overrightarrow{OM}$, $\overrightarrow{ON}$ and $\overrightarrow{OP}$ in component form. *[3 marks]*

(b) Find $\overrightarrow{MP} \times \overrightarrow{MN}$. *[4 marks]*

(c) **Hence**,

(i) calculate the area of the triangle MNP;

(ii) show that the line (AG) is perpendicular to the plane MNP;

(iii) find the equation of the plane MNP. *[7 marks]*

(d) Determine the coordinates of the point where the line (AG) meets the plane MNP. *[6 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

13. *[Maximum mark: 19]*

Let $f(x)=\dfrac{a+b\mathrm{e}^x}{a\mathrm{e}^x+b}$, where $0<b<a$.

(a) Show that $f'(x)=\dfrac{(b^2-a^2)\mathrm{e}^x}{(a\mathrm{e}^x+b)^2}$. *[3 marks]*

(b) **Hence** justify that the graph of f has no local maxima or minima. *[2 marks]*

(c) Given that the graph of f has a point of inflexion, find its coordinates. *[6 marks]*

(d) Show that the graph of f has exactly two asymptotes. *[3 marks]*

(e) Let $a=4$ and $b=1$. Consider the region R enclosed by the graph of $y=f(x)$, the y-axis and the line with equation $y=\dfrac{1}{2}$.

Find the volume V of the solid obtained when R is rotated through 2π about the x-axis. *[5 marks]*